This version, August 2017

Online Appendix for Influencing Connected Legislators

Abstract

In this appendix we present omitted proofs and table for "Influencing Connected Legislators."

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1 Proof of Lemma 1

Let $\boldsymbol{\varphi}$ be the n dimensional column vector of voting probabilities with ith element equal to φ_i. Define $\boldsymbol{\eta} : R^n \to R^n$ as $\boldsymbol{\eta}(\boldsymbol{\varphi}, \mathbf{s}) = \boldsymbol{\varphi} - \mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$, where $\mathbf{s} = (\mathbf{s_A}, \mathbf{s_B})$ and $\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$ is a column vector with ith element equal to $1/2 + \Psi\left(\omega(s_A^i) - \omega(s_B^i) + v^i q^i(\boldsymbol{\varphi}) + \phi \sum_j g_{i,j}(2\varphi_j - 1)\right)$, as defined in (5) in Section 3.1. The equilibrium probabilities $\boldsymbol{\varphi}(\mathbf{s})$ are defined as the solution of $\boldsymbol{\eta}(\boldsymbol{\varphi}^*, \mathbf{s}) = \mathbf{0}$. The fact that the solution of this system exists follows from Brouwer's fixed-point theorem as argued in Section 3. Since, by Assumption 1, $\Psi(\overline{v} + \phi + \omega(2W)) < 1/2$, the solution is interior in $(0,1)$. To show uniqueness of an equilibrium of the voting stage with policy motivated legislators for Ψ sufficiently small, let $\|x\|$ be the norm $\|x\| = \sum_i |x_i|$ for any $x \in R^n$. For any $\boldsymbol{\varphi}$ and $\boldsymbol{\varphi}'$, let $\boldsymbol{\varphi}_\xi = \boldsymbol{\varphi}' + \boldsymbol{\xi}(\boldsymbol{\varphi} - \boldsymbol{\varphi}')$ with $\xi \in [0,1]$ and $q_\xi^i(\boldsymbol{\varphi}_\xi)$ the directional derivative of $q_\xi^i(\boldsymbol{\varphi})$ at $\boldsymbol{\varphi}_\xi$. We can write:

$$\begin{aligned}
\left\|\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s}) - \mathbf{F}(\boldsymbol{\varphi}', \mathbf{s})\right\| &\leq \Psi \sum_i \left(\overline{v} \cdot \textstyle\int_0^1 q_\xi^i(\boldsymbol{\varphi}_\xi) d\xi + 2\phi \sum_j g_{i,j}\left(\varphi_j - \varphi_j'\right)\right) \\
&\leq \Psi\left(\overline{v} \cdot n \textstyle\int_0^1 \sum_j q_j^i(\boldsymbol{\varphi}_\xi)\left(\varphi_j - \varphi_j'\right) d\xi + 2\phi \sum_j \sum_i g_{i,j}\left(\varphi_j - \varphi_j'\right)\right) \\
&\leq \Psi(n\overline{v} + 2\phi\overline{g}) \sum_j \left(\varphi_j - \varphi_j'\right) \leq \Psi(n\overline{v} + 2\phi\overline{g}) \|\boldsymbol{\varphi} - \boldsymbol{\varphi}'\|
\end{aligned}$$

where we use the fact that $\left|q_j^i(\boldsymbol{\varphi}_\xi)\right| < 1$ for any $\xi \in [0,1]$, i and j; and $\sum_i g_{i,j} \leq \overline{g}$ for any j. For any $\eta < 1$, we therefore have $\left\|\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s}) - \mathbf{F}(\boldsymbol{\varphi}', \mathbf{s})\right\| \leq \eta \cdot \|\boldsymbol{\varphi} - \boldsymbol{\varphi}'\|$ for Ψ sufficiently small. We can therefore conclude that there is a Ψ_1 such that $\mathbf{F}(\boldsymbol{\varphi}, \mathbf{s})$ is a contraction in $[0,1]$ with a unique fixed-point in $(0,1)$ for $\Psi \leq \Psi_1$.

We now turn to the derivatives of the voting probabilities. The implicit function theorem implies that the solution φ_i is differentiable in s_A^j at $\mathbf{s}_A, \mathbf{s}_B$ if $(D\boldsymbol{\eta})_\varphi$ is invertible in a neighborhood of $(\mathbf{s}_A, \mathbf{s}_B, \boldsymbol{\varphi}(\mathbf{s_A}, \mathbf{s_B}))$, where $\boldsymbol{\varphi}(\mathbf{s_A}, \mathbf{s_B})$ solves $\boldsymbol{\eta}(\boldsymbol{\varphi}, \mathbf{s}_A, \mathbf{s}_B) = \mathbf{0}$ (the expression $(D\boldsymbol{\eta})_{\boldsymbol{\varphi}}$ represents the Jacobian of $\boldsymbol{\eta}$ with respect to $\boldsymbol{\varphi}$). It is easy to verify that $(D\boldsymbol{\eta})_{\boldsymbol{\varphi}} = \left[I - \Psi\widetilde{G}\right]$, where $\widetilde{G}$ is a $n \times n$ matrix with i,j element equal to $\widetilde{g}_{i,l} = \left(2\phi g_{i,l} + v^i q_l^i(\boldsymbol{\varphi})\right)$. It is easy to see that $\left[I - \Psi\widetilde{G}\right]^{-1}$ exists for Ψ sufficiently small. The Jacobian of φ with respect to s_A^j is:

$$D_j[\varphi] = \Psi \cdot \omega'(s_A^j)\left[I - \Psi\widetilde{G}\right]^{-1} \mathbf{1}_j,$$

where $\mathbf{1}_j$ is a n-dimensional vector equal to zero except at the jth dimension in which it is equal to one. As $\Psi \to 0$, the jth coordinate of $\omega'(s_A^j)\left[I - \Psi\widetilde{G}\right]^{-1} \mathbf{1}_j$ converges to $\omega'(s_A^j) > 0$, and the other coordiantes to zero. We conclude that there is a Ψ_2 such that for $\Psi < \Psi_2$, $\sum_i \partial\varphi_i / \partial s_A^j = D_j[\varphi]^T \cdot \mathbf{1} > 0$.

To verify concavity with respect to $\mathbf{s}_A$, let $D^2\varphi_i$ be the Hessian of φ_i. Consider first its diagonal entries $\partial^2\varphi_i/\partial s_A^j \partial s_A^j$ for any j. We can write:

$$\frac{\partial^2\varphi_i}{\partial s_A^j \partial s_A^j} = \Psi \left[\begin{array}{c} \frac{\partial^2 \omega(s_A^i)}{\partial s_A^j \partial s_A^j} + 2\phi \sum_l g_{i,l} \frac{\partial^2 \varphi_l}{\partial s_A^j \partial s_A^j} \\ +v^i \sum_l \left(\sum_k q_{lk}^i(\boldsymbol{\varphi}) \left(\frac{\partial \varphi_l}{\partial s_A^j} \right) \left(\frac{\partial \varphi_k}{\partial s_A^j} \right) + q_l^i(\boldsymbol{\varphi}) \frac{\partial^2 \varphi_l}{\partial s_A^j \partial s_A^j} \right) \end{array} \right]. \tag{1}$$

We can write:

$$\left[I - \Psi \widetilde{G} \right] D_{j,j}^2 [\boldsymbol{\varphi}] = \Psi \omega''(s_A^j) \left(\mathbf{1}_j + \Psi^2 \frac{\left(\omega'(s_A^j) \right)^2}{\omega''(s_A^j)} V \left(\mathbf{z}^j \right)^T D^2 q^i(\boldsymbol{\varphi}) \left(\mathbf{z}^j \right) \right), \tag{2}$$

where $D_{j,j}^2 [\boldsymbol{\varphi}] = (\frac{\partial^2 \varphi_1}{\partial s_A^j \partial s_A^j}, ..., \frac{\partial^2 \varphi_n}{\partial s_A^j \partial s_A^j})^T$, the $n \times n$ matrix $D^2 q^i(\boldsymbol{\varphi})$ is the Hessian of $q^i(\boldsymbol{\varphi})$, and $\mathbf{z}^j = \left[I - \Psi \widetilde{G} \right]^{-1} \mathbf{1}_j$. The vector $D_{j,j}^2 [\boldsymbol{\varphi}]$ exists if $\left[I - \Psi \widetilde{G} \right]$ is invertible: a property that, as shown above, is verified if $\Psi \leq \Psi_2$. Since $\left(\omega'(s_A^l) \right)^2 / \omega''(s_A^l)$ is bounded for any feasible s_A^l, and $\mathbf{z}^j$ is a bounded column vector, we have:

$$D_{j,j}^2 [\boldsymbol{\varphi}] = \Psi \left[I - \Psi \widetilde{G} \right]^{-1} \omega''(s_A^j) \left(\mathbf{1}_j + o(\Psi^2) \right), \tag{3}$$

where $o(\Psi^2)$ converges to zero as $\Psi \to 0$ at the speed of Ψ^2. As $\Psi \to 0$, the jth coordinate of $\omega''(s_A^j) \left[I - \Psi \widetilde{G} \right]^{-1} \mathbf{1}_j$ converges to $\omega''(s_A^j) < 0$, and the other coordiantes to zero. We conclude that there is a Ψ_3 such that for $\Psi < \Psi_3$, $\sum_i \partial^2 \varphi_i / \partial s_A^j \partial s_A^j = D_{j,j}^2 [\boldsymbol{\varphi}] \cdot \mathbf{1} < 0$. We conclude that the diagonal of the Hessian of $\sum_i \varphi_i$ has all strictly negative values. Following the same steps as above it we can also show that for any ε there is a Ψ_3 such that the absolute values of the off diagonal elements of the Hessian of $\sum_i \varphi_i$ are lower than ε for $\Psi \leq \Psi_3$. This implies that there is a Ψ^* such that $\sum_i \varphi_i$ is increasing and strictly concave in s_A^j and $\mathbf{s}_A$ for $\Psi \leq \Psi^*$. ■

2 Proof of Lemma 3.1

The fact that all agents of the same type have the same Katz-Bonacich centrality is immediate from the definition. We can write:

$$b_i(\phi^*, G^T) = 1 + \phi^* \sum_{l=1}^n g_{l,i} \cdot b_l(\phi^*, G^T) = 1 + \phi^* \sum_{\tau=1}^m n_\tau h_{\tau,\iota(i)} \cdot \overline{b}_\tau$$

where $\overline{b}_\tau$ be the Katz-Bonacich centrality of an agent of type τ. Since, again, $b_i(\phi^*, G) = \overline{b}_{\iota(i)}$, we have: $\overline{b}_{\iota(i)} = 1 + \phi^* \sum_{\tau=1}^m \widetilde{h}_{\tau,\iota(i)} \cdot \overline{b}_\tau$ where $\widetilde{h}_{\tau,\iota(i)} = n_\tau h_{\tau,\iota(i)} = \alpha_\tau h_{\tau,\iota(i)} / (\sum_{l=1}^m \alpha_l h_{\tau,l})$, since

$\sum_{l=1}^{m}\alpha_l h_{\tau,l}=\sum_{j=1}^{n}g_{i,j}/n=1/n$. We therefore have that $\overline{\mathbf{b}}=\left[I+\phi^*\widetilde{H}^T\right]^{-1}\cdot\mathbf{1}$, implying that $b_i(\phi^*,G)$ is defined by (31) in Section 7.3 as stated. ■

3 Proof of Lemma 3.2

We first note that by Assumption 1 $\varphi_j\le\overline{\varphi}$, $\varphi_j\ge\underline{\varphi}$ for some $\overline{\varphi}$ and $\underline{\varphi}$ in $(0,1)$, any legislator j and any $\mathbf{s}_A,\mathbf{s}_B$. Given this, we proceed in two steps.

Step 1. We prove here that $\lim\limits_{n\to\infty}q^{n,j}=0$ for all $j=1,...,n$. Consider the pivot probability of a player j of type i. There are two cases to consider.

Case 1.1. Suppose first that $\alpha_i^n\to\alpha_i>0$. Let M_{-i}^n be the profile of votes of all types different from i. Let P_i^n be the probability that there is a profile of votes M_{-i}^n such that j can be pivotal for some profile $m_i^{-j,n}$ of players of type i different than j. Let $p_j^n(M_{-i}^n)$ be the probability of $m_i^{-j,n}$ such that j is pivotal given M_{-i}^n and let $\overline{p}_j^n=\max_{M_{-i}^n}p_j(M_{-i}^n)$. Associated to $\overline{p}_j^n$ there is a number $\widehat{l}_j^n$ of legislators of type i that must vote for A in order for j to be pivotal. Let $\eta_j^n=\widehat{l}_j^n/(n_i-1)$ the share of types i other than j that are needed to make j pivotal. If $\eta_j^n\to1$ or $\eta_j^n\to0$ then $\overline{p}_j^n$ converges to zero, so j's pivot probability converges to zero. Assume $\eta_j^n\to\eta_j\in(0,1)$. Given this, j's pivot probability can be bounded above as follows. To keep the formulas simple, let $z_i(n)=\alpha_i n-1$

$$
\begin{aligned}
\lim_{n\to\infty}q^{n,i} &\le P_i^n\cdot\lim_{n\to\infty}b(\eta_j^n z_i(n);z_i(n),\varphi^i)\\
&\le \lim_{n\to\infty}\begin{pmatrix} z_i(n)\\ \\ \eta_j^n z_i(n)\end{pmatrix}\left(\left(\varphi^i\right)^{\eta_j^n z_i(n)}\cdot\left(1-\varphi^i\right)^{\left(1-\eta_j^n\right)z_i(n)}\right)\\
&\le \lim_{n\to\infty}\frac{\left(\sqrt{2\pi z_i(n)}\cdot\left(z_i(n)\right)^{z_i(n)}e^{-z_i(n)}\right)\cdot\left(\left(\varphi^i\right)^{\eta_j^n z_i(n)}\cdot\left(1-\varphi^j\right)^{\left(1-\eta_j^n\right)z_i(n)}\right)}{\left(\begin{array}{c}\left(\sqrt{2\pi\eta_j^n z_i(n)}\cdot\left(\eta_j^n z_i(n)\right)^{\eta_j^n z_i(n)}e^{-\eta_j^n z_i(n)}\right)\\ \cdot\sqrt{2\pi(1-\eta_j^n)z_i(n)}\cdot\left(\left(1-\eta_j^n\right)z_i(n)\right)^{\left(1-\eta_j^n\right)z_i(n)}e^{-\left(1-\eta_j^n\right)z_i(n)}\end{array}\right)}\\
&= \lim_{n\to\infty}\frac{1}{\left(\left(\sqrt{2\pi\eta_j^n}\right)\cdot\sqrt{(1-\eta_j^n)}\right)}\cdot\left(\frac{\left(\left(\varphi^i\right)^{\eta_j^n}\cdot\left(1-\varphi^j\right)^{1-\eta_j^n}\right)}{\left(\eta_j^n\right)^{\eta_j^n}\left(1-\eta_j^n\right)^{1-\eta_j^n}}\right)^{z_i(n)}\cdot\frac{1}{\sqrt{z_i(n)}}\\
&\le \frac{1}{\left(\left(\sqrt{2\pi\eta_j}\right)\cdot\sqrt{(1-\eta_j)}\right)}\lim_{n\to\infty}\frac{1}{\sqrt{z_i(n)}}=0,
\end{aligned}
$$

where the third inequality follows from the Stirling formula and the last follows from the fact that $\eta_j^n \in \arg\max_\varphi \left((\varphi)^{\eta_j^n} (1-\varphi)^{1-\eta_j^n} \right)$.

Case 1.2. Consider now that case in which $\alpha_i^n \to 0$. Let M_{-jk}^n the profile of votes of: 1) all types i but different than agent j; and 2) of all other types $t \neq i, k$, where k is a type such that $\alpha_k^n \to \alpha_k > 0$. Let P_{-jk}^n be the probability that there is a profile of votes M_{-jk}^n such that j can be pivotal for some profile m_k^n of players in k. Let $p_j^n(M_{-jk})$ be the probability of m_k^n such that j is pivotal given M_{-jk} and let $\overline{p}_j^n = \max_{M_{-jk}^n} p_j(M_{-jk}^n)$. As above the pivot probability $q^{n,i}$ can be bounded above by $P_{-jk}^n \cdot \overline{p}_{jk}^n$. Proceeding as in the previous case, we can prove that this upper bound converges to zero as $n \to \infty$, implying the result.

Step 2. Consider now $\sum_j \left| q_j^{n,i} \right|$. For any two distinct legislators i and j, let N^{-ij} and $\boldsymbol{\varphi}^{-ij}$ be, respectively, the set of all legislators except i and j and the associated vector of probabilities of choosing P. Let moreover $S(N^{-i}, s)$ be the set of all s-combinations of N^{-ij}. We have that for any $j \neq i$, $q^{n,i} = \varphi_j E_n + (1-\varphi_j) F_j$ where:

$$\begin{aligned} E_n &= \sum_{A \in S(N^{-ij}, qn-2)} \prod_{k \in A} \varphi_k^{-ij} \cdot \prod_{l \in A^C} (1-\varphi_l^{-ij}) \\ F_n &= \sum_{A \in S(N^{-ij}, qn-1)} \prod_{k \in A} (\varphi_k^{-ij}) \cdot \prod_{l \in A^C} \left(1-\varphi_l^{-ij}\right) \end{aligned}$$

We can therefore write: $q_j^{n,i} = (E_n - F_n)$. From Step 1 we know that $q^{n,i} \to 0$ as $n \to \infty$ for all i. It follows from (5) in Section 3.1, that $\varphi_i \to 1/2$ for all legislators. This implies that, for all i, $|E_n - F_n|$ can be bounded above by: $K_n = \Theta \begin{pmatrix} n \\ qn \end{pmatrix} \left((1+\delta)/2 \right)^n$ where $\Theta > 1$, and $\delta > 0$ is a parameter that can be chosen arbitrarily close to 0 for n sufficiently large. It follows that $\sum_j \left| q_j^{n,i} \right|$ is bounded above by nK_n. Using again the Stirling formula we have:

$$\begin{aligned} \lim_{n\to\infty} nK_n = \lim_{n\to\infty} & \frac{n\Theta\sqrt{2\pi n} n^n e^n}{\left[\begin{array}{c} \sqrt{2\pi qn}\,(qn)^{qn}\, e^{qn} \cdot \\ \sqrt{2\pi\,(1-q)\,n}\,[(1-q)\,n]^{(1-q)n}\, e^{(1-q)n} \end{array} \right]} \left((1+\delta)/2 \right)^n \\ = \frac{\Theta}{\sqrt{2\pi q(1-q)}} \lim_{n\to\infty} & \left(n^{1/2} \left(\frac{2 \cdot q^q (1-q)^{1-q}}{1+\delta} \right)^{-n} \right) = \frac{\Theta}{\sqrt{2\pi q(1-q)}} \lim_{n\to\infty} \left(n^{1/2} (1-\epsilon)^n \right) \end{aligned}$$

for some $\epsilon > 0$, where the last equality follows from the fact that, since δ is arbitrarily small,

$2 \cdot q^q (1-q)^{1-q} / (1+\delta) > 1$ for any $q \in (1/2, 1)$. Since $\lim_{n\to\infty} \left(n^{1/2} (1-\epsilon)^n\right) = 0$, we have that $\sum_j \left|q_j^{n,i}\right|$ converge to zero. ■

4 Proof of the result stated in Section 5.4

Let us define $\beta_l^n(s_A, s_B)$ as the probability that threshold l is passed for $l = 1, ..., J$, $\beta_l^n(s_A, s_B) = \Pr\left(\sum_i \chi_i^n(A) > z_l \left| \mathbf{s}_A, \mathbf{s}_B\right.\right)$. With preferences that depend on reaching the threshold z_j, interest group A's expect utility can be written as: $W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B) = u_0 + \sum_{l=0}^{J} (u_l - u_{l-1}) \beta_l^n(\mathbf{s}_A, \mathbf{s}_B)$. The equilibrium contributions are characterized by the first order necessary condition of:

$$\max_{(\mathbf{s}_A, \mathbf{s}_B) \in S} W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B). \tag{4}$$

The necessary condition of the corresponding Lagrangian with respect to s_A^j where j is an agent of type i:

$$\frac{\partial W_n^{\mathbf{z},\mathbf{u}}(\mathbf{s}_A, \mathbf{s}_B)}{\partial s_A^j} = \sum_k \left(\sum_l (u_l - u_{l-1}) \frac{\partial \beta_l^n}{\partial \varphi_k}\right) \cdot \frac{\partial \varphi_k^n}{\partial s_A^j} = \lambda^n \tag{5}$$

where $\partial\beta_l^n/\partial\varphi_k$ and $\partial\varphi_k^n/\partial s_A^j$ are the derivatives of $\beta_l^n(s_A, s_B)$ and $\varphi_k^n(s_A, s_B)$ with respect to, respectively, φ_k^n and $s_A^{n,j}$ evaluated at $\widetilde{\mathbf{s}}$ and λ^n is chosen to satisfy the budget constraint. It is easy to verify that $\partial\beta_l^n/\partial\varphi_k$ is equal to the probability that legislator k is "pivotal" in having threshold l passed, that is $\partial\beta_l^n/\partial\varphi_k = \beta_l^{-k,n}$ where $\beta_l^{-k,n} = \Pr\left(\sum_{i\neq k} \chi_i^n(A) = z_l \left| \mathbf{s}_*, \mathbf{s}_*\right.\right)$. We can rewrite (5) as:

$$\frac{\sum_{k=1}^n (R_k^n/R_1^n) \cdot \partial\varphi_k^n/\partial s_A^j}{\sum_{k=1}^n (R_k^n/R_1^n) \cdot \partial\varphi_k^n/\partial s_A^l} = 1,$$

where $R_k^n = \sum_l \left[(u_l - u_{l-1}) \cdot \partial\beta_l^n/\partial\varphi_k\right]$. Note that, by Lemma 3.2, $q_n^i \to 0$ as $n \to \infty$, so by (5) in section 3.1 we must have that the probability that i votes for A is $\varphi_{i,n} \to 1/2$ as $n \to \infty$. This implies that $\beta_l^{-k}/\beta_1^{-k} \to 1$ and so $R_j^n/R_1^n \to 1$ for any $j = 1, .., m$. It follows that

$$\begin{aligned}
\lim_{n\to\infty} \frac{\sum_{k=1}^n (R_k^n/R_1^n) \cdot \partial\varphi_k^n/\partial s_A^j}{\sum_{k=1}^n (R_k^n/R_1^n) \cdot \partial\varphi_k^n/\partial s_A^l} &= \lim_{n\to\infty} \frac{\sum_{k=1}^n \partial\varphi_k^n/\partial s_A^j}{\sum_{k=1}^n \partial\varphi_k^n/\partial s_A^l} = \lim_{n\to\infty} \frac{b_j^{\mathcal{M}}(\phi^*, V, G^T)\omega'(s_A^j)}{b_l^{\mathcal{M}}(\phi^*, V, G^T)\omega'(s_A^l)} \\
&= \lim_{n\to\infty} \frac{b_j(\phi^*, G^T)}{b_l(\phi^*, G^T)} \frac{\omega'(s_A^j)}{\omega'(s_A^l)} = 1 \quad \forall j, l
\end{aligned}$$

where the second equality follows from the analysis of $D_i[\boldsymbol{\varphi}]^T \cdot 1$ in Section 7.2 and $\left(b_i(\phi^*, G^T)\right)_{i=1}^n$ are the limit Katz-Bonacichs. We conclude that for a large n, we have $\frac{\omega'(s_A^j)}{\omega'(s_A^l)} \simeq \frac{b_l(\phi^*, G^T)}{b_j(\phi^*, G^T)}$, or $b_j(\phi^*, G^T)\omega'(s_A^j) \simeq \lambda$ for all $j = 1, ..., n$. Assuming log utility as in Section 4 of the paper, we have $s_A^j \simeq b_j(\phi^*, G^T)$ for all $j = 1, ..., n$. ■

Table A.3 Top 50 educational institutions by number of alumni

	School name	Total number of alumni	Congress				
			109	110	111	112	113
1	California State University	43	8	9	8	9	9
2	Harvard University	36	3	6	10	7	10
3	University of California Berkeley	33	7	8	7	7	4
4	University of California Los Angeles	32	5	6	8	7	6
5	University of Florida	32	6	6	8	6	6
6	George Washington University	31	7	8	6	6	4
7	University of North Carolina	30	6	7	6	6	5
8	University of Texas at Austin	30	6	5	5	7	7
9	Georgetown University	29	7	7	4	4	7
10	University of Georgia	27	3	4	5	7	8
11	Stanford University	26	6	5	5	5	5
12	Southern Methodist University	25	5	5	5	5	5
13	University of Southern California	25	5	7	6	5	2
14	Texas A&M University	22	5	5	5	4	3
15	Yale University	22	3	4	6	5	4
16	University of Michigan	22	5	4	3	4	6
17	Duke University	21	4	4	3	5	5
18	New York University	21	4	4	4	4	5
19	Louisiana State University	20	4	5	4	4	3
20	University of Wisconsin	20	3	6	6	2	3
21	Florida Agricultural and Mechanical University	18	4	4	4	3	3
23	Ohio State University	18	4	4	3	3	4
24	Notre Dame University	16	3	4	4	3	2
25	University of Pittsburgh	16	6	4	2	2	2
26	Cornell University	15	2	2	3	5	3
27	Columbia University	15	3	3	3	3	3
28	University of Minnesota	15	3	4	3	2	3
29	Northwestern University	13	4	3	2	3	1
30	University of Oregon	13	3	3	2	2	3
31	University of Illinois	13	1	1	2	4	5
32	University of California Davis	13	1	3	3	3	3
33	London School of Economics	12	3	3	2	2	2
34	University of Alabama	12	3	3	2	2	2
35	Brigham Young University	11	2	2	2	2	3
36	California Polytechnic State University	11	3	2	1	2	3
37	Washington and Lee University	11	3	2	2	2	2
38	Case Western Reserve University	11	3	4	3	1	0
39	Marshall Scholar	11	2	2	2	2	3
40	St. John's University	10	3	2	2	2	1
41	Texas Tech University	10	2	2	2	2	2
42	Dartmouth College	10	2	2	2	2	2
43	Pacific Lutheran University	10	2	2	2	2	2
44	State University of New York	10	3	4	3	0	0
45	West Point Military Academy	10	1	1	2	3	3
46	Cleveland State University	10	2	2	3	2	1
47	Arizona State College of Law	10	1	2	2	2	3
48	Florida State University	10	0	2	3	2	3
49	Princeton University	10	0	1	3	4	2
50	Massachusetts Institute of Technology	9	2	2	2	2	1